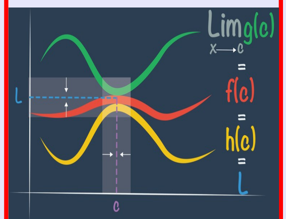
Math 261

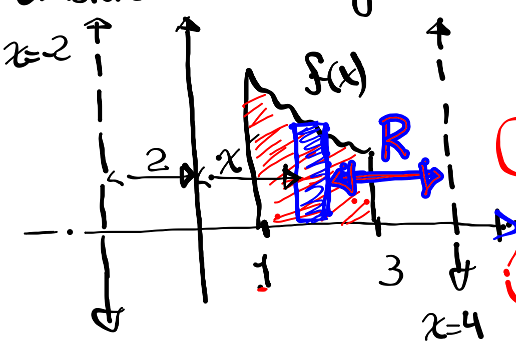
Spring 2021

Lecture 54



Class QZ 18.

Consider the region below



$x+R=4$

$R=4-x$

Set-up only

Volume rotated about

① x-axis $\int_1^3 \pi[f(x)]^2\,dx$

② $x=-2$ $\int_1^3 2\pi(2+x)f(x)\,dx$

③ $x=4$

$V=\int_1^3 2\pi(4-x)f(x)\,dx$

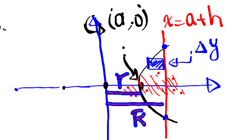
Rotate the region enclosed by

$x^2 - y^2 = a^2$, $x = a+h$ $\quad a>0, h>0$

about y-axis.

$\frac{x^2}{a^2} - \frac{y^2}{a^2} = 1$

hyperbola



Washer Method

$R = a+h$

$r = x$

$$V = 2\int_0^{\sqrt{2ah+h^2}} \pi\left[R^2 - r^2\right]dy$$

$x = a+h$

$x^2 - y^2 = a^2$

$(a+h)^2 - y^2 = a^2$

$\cancel{a^2} + 2ah + h^2 - y^2 = \cancel{a^2}$

$2ah + h^2 = y^2$

$y = \sqrt{2ah+h^2}$

$x^2 - y^2 = a^2$

$x^2 = a^2 + y^2$

$$V = 2\pi \int_0^{\sqrt{2ah+h^2}} \left[(a+h)^2 - x^2\right]dy$$

$$= 2\pi \int_0^{\sqrt{2ah+h^2}} \left[\cancel{a^2} + 2ah + h^2 - \cancel{a^2} - y^2\right]dy$$

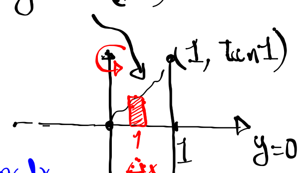
check it out.

$$= \boxed{\frac{4\pi}{3}\left(2ah+h^2\right)^{3/2}}$$

Rotate the region bounded by

$y=0$, $x=1$, and $y = \tan(x^2)$

about y-axis.



$$V = \int_0^1 2\pi x \tan x^2 \, dx$$

$u = x^2 \quad du = 2x\,dx$

$x=0 \quad u=0$

$x=1 \quad u=1$

$$= \pi \int_0^1 \tan u \, du$$

$w = \cos u$

$dw = -\sin u \, du$

$$= \pi \int \frac{\sin u}{\cos u} du$$

$u=0 \quad w=1$

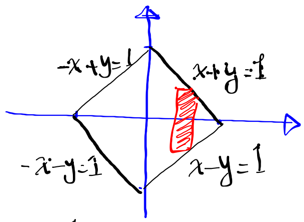
$u=1 \quad w=\cos 1$

$$= -\pi \int_1^{\cos 1} \frac{1}{w} dw = -\pi \ln w \Big|_1^{\cos 1}$$

$$= -\pi \ln(\cos 1) \approx \boxed{\pi \cdot .616}$$

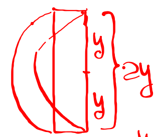
$$\approx \boxed{.616\pi}$$

Draw the region enclosed by a square with vertices at
(1,0), (0,1), (-1,0), (0,-1)



Cross-Sections $\perp$ x-axis

Cross-Sections are Semicircles.



$y=1-x$

$$V=2\int_0^1 \frac{\pi R^2}{2}\,dx$$

$$=\pi\int_0^1 (2y)^2\,dx$$

$$=4\pi\int_0^1 (1-x)^2\,dx$$

$u=1-x$ $x=0$ $u=1$

$du=-dx$ $x=1$ $u=0$

$-du=dx$

$$=-4\pi\int_1^0 u^2\,du$$

$$=4\pi\int_0^1 u^2\,du=4\pi\frac{u^3}{3}\Big|_0^1=\boxed{\frac{4\pi}{3}}.$$

Find f_{ave} for $f(x)=x\sin x^2$ over $[0,10]$

$$f_{ave}=\frac{1}{b-a}\int_a^b f(x)\,dx$$

$$=\frac{1}{10}\int_0^{10} x\sin x^2\,dx$$

$u=x^2$

$du=2x\,dx$

$\frac{du}{2}=x\,dx$

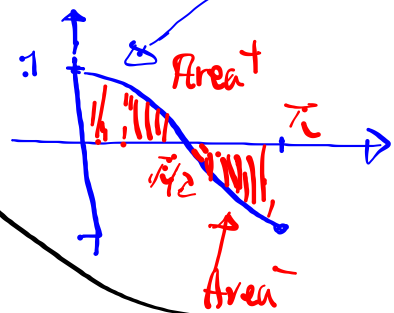
$$=\frac{1}{10}\int_0^{100}\sin u\,\frac{du}{2}$$

$$=\frac{1}{20}(-\cos u)\Big|_0^{100}=\boxed{\frac{1-\cos 100}{20}}$$

without using Calc

find f_{ave} for $f(x)=\cos x$ on $[0,\pi]$

$=\boxed{0}$



$$f_{ave}=\frac{1}{\pi-0}\int_0^{\pi}\cos x\,dx$$

$$=\frac{1}{\pi}\sin x\Big|_0^{\pi}=\frac{1}{\pi}\left[\sin\pi-\sin 0\right]=\boxed{0}$$